On Statistical Quality Control of High Strength Concrete

—Some Experience in the STRONG ROOM Construction at Technical Research Institute, OHBAYASHI-GUMI, LTD.—

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Synopsis

There were following individualities in the construction of the STRONG ROOM inscribed on the title.

(a) The structural design strength was so high as 300 kg/cm^2 (4254 psi),

(b) On account of massive concrete, the cement quantity was reduced to the minimum in mix design,

(c) Therefore, its consistency became such dry one as of 8.0 cm SLUMP,

(d) Moreover, those dry consistency concrete was pumped up all through the construction work.

With the progress of construction, the actual condition of concrete quality such as strength and consistency, these changes in transporting, and pumpability of concrete were investigated.

As the result, in spite of difficult condition of pumping up dry consistency concrete, the construction work was satisfactorily done and the expected quality level of concrete was almost ensured.

In the second half of this report, statistical contemplation was referred to about the sample datum and sampling methods. The number of sampled test pieces was much larger than the one prescribed in Japanese Industrial Standard (JIS A5308), Statistical meaning was well defined about calculated results from the samples, and difference of precision of population estimating was pointed out between in the adopted method and in JIS method.

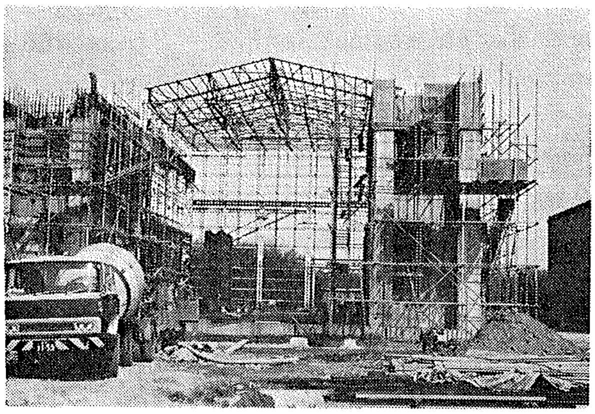


Photo-1 a scene under construction of STRONG ROOM

1. METHODS AND PROCEDURE OF THE INVESTIGATION

1.1. Sampling Method

Sampling frequency provided in JIS A5308 is such that a test should be carried out per 150 m^3 concrete volume and in each test three test pieces are to be sampled. The sampling frequency adopted was about 10 times larger than that in JIS method, and a test was carried out per 10～15 m^3 (agitator truck 3～5). At the same time, ready-mixed concrete plant made their own quality control as maker's side according to JIS method.

1.2. Investigation Procedure

The construction term was from March, 1970 to March, 1971. During this period there were 14 concrete depositing days and 1900 m^3 of high strength concrete was placed in the total. On every depositing day, sampling was carried out as frequently as possible and total number of sampled truck agitators became 157. The influence of transport on the properties of fresh concrete and possibility of pumping up dry consistency concrete were studied at the start of the construction work. On the basis of those result mixing ratio was fixed as followe.

W/C (%)	S/A (%)	Water (kg)	Cement (kg)	Sand (kg)	Gravel (kg)	Admixture Pozolis No5 L (kg)
45.5	41.1	143	315	787	1139	7.12

Table-1 Mixing proportion per $1m^3$ (by weight)

1.3. Mixing Ratio

Table-1 shows the mixing ratio of the high strength concrete. The objective slump value and the air content were 8 ± 2.0 cm and $4 \pm 1.0\%$ respectively (at the discharge). The correction of strength for temperature was not done because of mass concrete and this ratio had been kept through out the construction work.

As for materials, gravel from the river Arakawa (F.M. 6.91) and sand from the river Watarase (F.M. 2.98) were used as aggregate. Cement in use was Chichibu normal portland cement of fineness 3130 cm^2/g.

No.	Slump (cm)			Air content %			Concrete temp. ℃		
	A plant	B site	B−A	A plant	B site	B−A	A plant	B site	B−A
1	14.1	10.6	−3.5	5.0	4.4	−0 6	18.0	20.0	2.0
2	15.5	12.0	−3.5	4.7	4.3	−0 4	18.2	20.0	1.8
3	12.1	8.7	−3.4	4.5	4.9	−0 4	21.0	22.0	1.0
Ave.	13.9	10.4	−3.5	4.7	4.5	−0 2	19.1	20.6	1.5

Table-2 Influence of the transportation on fresh concrete (checking up Apr. 1970)

2. RESULT OF THE SURVEY

2.1. Influence of Transport

It took about 30 to 35 minutes to carry ready mixed concrete from the batcher plant to the site. There were some changes in the properties of fresh concrete during the transporting as shown in Table-2. Half an hour transporting brought 3∼4 cm slump loss, and so it was open to reason that the objective slump value at the concrete plant should be 12 ± 2.0 cm.

date	Pumped dn volume (m^3/hr)	pump		converted length of pipe-line (m)	pipe diameter (inch)	type of pump
		pressure (kg/cm^2)	cycle (rpm)			
4/23	34±1.8	50∼75	26±3.3	93	4′	IHI PTF40TP
9/18	35±0.5	75∼85	31±1.0	81	5′	IHI PT40P

Table-3 Pumpability of high strength concrete of dry consistency

2.2. Pumpability

Table-3 shows the ordinary pumpability of dry consistency concrete. At the beginning it was considered almost impossible to pump up such concrete. However, it truned out practicable without blocking up by making some modifi-

date	Sample size n	Slump (cm)			Air content (%)			7 days strength (kg/cm^2)			28 days strength (kg/cm^2)		
		x̄	σ	V (%)	x̄	σ	V (%)	x̄	σ	V (%)	x̄	σ	V (%)
70′.6.2	14	12.2	3.6	30	2.9	0.4	14	227	20	9	311	20	6
7.8	14	8.6	2.0	23	2.9	0.6	19	243	15	6	326	30	9
7.25	11	9.9	2.5	25	3.3	0.5	14	195	40	20	293	15	5
9.18	15	10.5	2.6	24	3.1	0.6	20	227	21	9	358	23	7
9.28	9	8.8	4.1	46	3.7	0.9	24	231	24	10	322	35	11
11.4	12	12.8	2.4	19	4.0	0.5	13	208	14	7	318	16	5
12.25	7	15.8	1.6	10	4.6	0.4	9	265	30	11	332	18	5
12.30	6	12.7	2.2	17	4.5	0.5	11	204	24	12	324	18	6
71′.2.2	12	11.1	2.3	21	3.6	0.5	14	243	18	7	339	17	5
2.11	10	12.2	2.2	18	6.1	0.3	4	220	18	8	339	17	5
2.19	9	9.9	3.9	40	4.9	0.7	15	254	33	13	356	17	5
3.2	11	8.3	1.6	19	3.9	0.4	10	259	24	9	361	20	6
3.10	6	8.4	1.3	16	4.8	0.1	2	241	15	6	341	9	3
3.17	11	8.1	1.2	14	3.9	0.4	11	245	22	9	342	22	6

Table-4 Sampling result in each concrete placing day cations of S/A ratio and cement quantity. Efficiency of concreting was for the most part satisfactory one. Converted horizontal length of the pipe line was calculated in accordance with the recommendation of the concrete pump sub committee of A.I.J.

2.3. Dispersion of Strength, Consistency, and etc.

Sampling result on every concreting day was shown in Table-4, but it was impossible due to lack of space to show raw datum of the samples and control charts. The mean values and the standard deviations of the 4 weeks strength, slump and air content all through the construction were as follows.

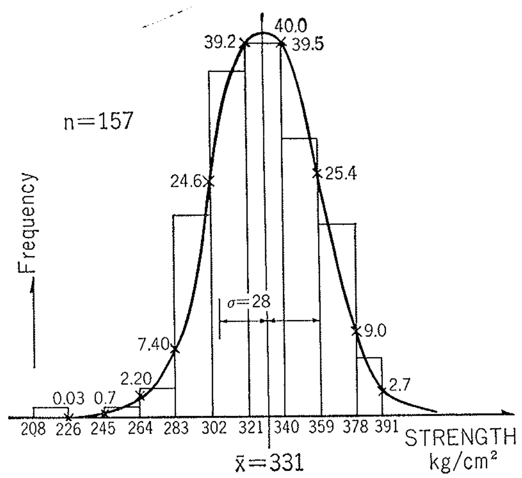


Fig.-1 Histogram of 28 days strength and normal distribution curve N(331,28²)

Compressive strength:	Slump value:	Air content:
$\bar{\chi} = 331\ \text{kg/cm}^2$	$\bar{\chi} = 10.6\ \text{cm}$	$\bar{\chi} = 3.8\%$
$\sigma = 28\ \text{kg/cm}^2$	$\sigma = 3.1\ \text{cm}$	$\sigma = 0.99\%$
$\nu = 8.5\%$	$\nu = 30\%$	$\nu = 26\%$

Fig.-1 shows the histogram of compressive strength (age 28 days). Null hypothesis on its goodness of fit for the normal distribution N(331,28²), were rejected at 5% level of significance as the result of Chi-square test ($\chi_0^2 = 10.9$), and so it may be interpreted reasonably that samp les form N(331,28²).

The whole variance of the strength were divided into variance within a day and between days. Each standard deviation was calculated respectively as follows.

* standard deviation within a day

$$\sigma_w = 19.8\ \text{kg/cm}^2 \qquad (V = 5.9\%)$$

* standard deviation between days

$$\sigma_b = 19.0\ \text{kg/cm}^2 \qquad (V = 5.7\%)$$

* whole standard deviation

$$\sigma = \sqrt{19.8^2 + 19.0^2} = 27.6\ \text{kg/cm}^2 \quad (V = 8.3\%)$$

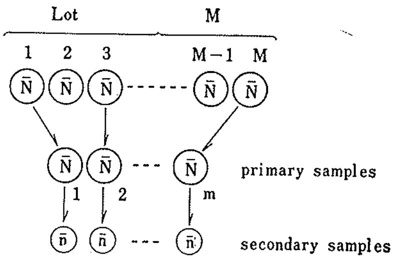
This whole standard deviation fits well with the value shown in Fig.-1.

As for the sampling result which was put into operation by the concrete plant, 355 kg/cm² was got as the mean concrete strength from 24 times' samplings and 12 kg/cm² as its standard deviation. Difference of the result between the user and the maker came mainly from the different conditions between the both laboratories and not from the sampling method as stated afterwards in detail.

3. STATISTICAL CONTEMPLATION OF THE RESULT

3.1. Application of Two Stage Sampling Theory

The precision of population mean estimation of every concrete placing day was calculated applying two stage sampling theory. As shown in Fig.-2, when m truckmixers are sampled from M as the primary sampling and from each, $\bar{n}$ increments are sampled as the secondary sampling,



$\bar{N}$: one truck mixer (primary sampling unit)

$\bar{n}$: the number of test cylinders (secondary sampling unit)

Fig.-2 Two stage sampling

date	concrete volume (m^3)	agitator truck		primary sampling ratio	secondary sampling ratio	error variance V($\bar{x}$)	standard error D($\bar{x}$)%
		total	sampled one				
6/2	246	53	14	0.26	1/1875	2.40	1.6
7/8	336	75	22	0.29	〃	2.98	1.7
7/25	234	49	11	0.22	〃	2.34	1.5
9/18	216	45	15	0.33	〃	2.73	1.7
9/28	84	16	9	0.56	〃	7.12	2.7
11/4	168	36	12	0.33	〃	2.06	1.4
12/25	83	16	7	0.44	〃	3.23	1.8
12/30	78	15	6	0.40	〃	5.13	2.3
2/2	83	20	12	0.60	〃	1.51	1.2
2/11	82	14	10	0.71	〃	1.53	1.2
2/19	66	12	9	0.75	〃	1.60	1.3
3/2	65	12	11	0.92	〃	0.94	1.0
3/10	69	14	6	0.43	〃	2.19	1.5
3/17	69	12	11	0.92	〃	0.99	1.0

Table-5 Precision of population mean estimation

the error variance of population mean estimation would be given by expression (1).

$$\nu(\bar{x}) = \frac{M-m}{M-1}\cdot\frac{\sigma_b^2}{m} + \frac{\bar{N}-\bar{n}}{\bar{N}-1}\cdot\frac{\sigma^2\omega}{m\bar{n}} + \sigma_R^2 + \frac{\sigma^2 M}{k} \quad \cdots\cdots (1)$$

$$\doteqdot \left(1-\frac{m}{M}\right)\cdot\frac{\sigma_b^2}{m} + \left(1-\frac{\bar{n}}{\bar{N}}\right)\cdot\frac{\sigma^2\omega}{m\bar{n}} + \sigma_R^2 + \frac{\sigma_M^2}{K} \quad \cdots\cdots (2)$$

where

σ_b^2: variance between primary sampling units

σ_ω^2: variance within primary sampling units

σ_R^2: precision of reduction

σ_M^2: precision of measurement

k: the number of analysis

According to the result of cooperation test carried by the Cement Association of Japan, experimental error between laboratories might be treated as 4.7% and inevitable error within the same laboratory as 1.9% and the error from the difference of testing apparatus containig its operation as 2.1%. We had obtained the value $(2.0\%)^2$ as the variance within a truck-mixer from the other survey. The error of reduction (preparation of samples) seems to be almost negligible. Therefore, σ_ω, σ_R, and σ_M can be determined as follows.

$\sigma_\omega = 2.0\%$, $\sigma_R = 0\%$, $\sigma_M = 1.9\%$,

Besides these values, putting given M, m, and σ_b of every concreting day into the equation (2), we can get the error variance and the standard error in each case as shown in Table-5. The error of estimation in the probability of 95% should be twice the standard error.

For example, if we want to design the sampling method of precision β in the probability of 95%, the following equation will be given.

$$\beta = 2\sqrt{\nu(\bar{x})} \quad \cdots\cdots (3)$$

Into this equation, put the known value of σ_b, σ_ω, and σ_M given from the preliminaly investigation and then M and $\bar{n}$ can be determind.

Assume that 150 m^3 concrete is placed in site and three test pieces of 15ϕ × 30 cm cylinder are sampled from one truck-mixer according to JIS A5308, and then the error variance of estimated population mean is thus.

$$\nu(\bar{x}) = \frac{50-1}{50-1}\cdot\frac{\sigma_b^2}{1} + \frac{566-3}{566-1}\cdot\frac{\sigma_\omega^2}{3} + \sigma_M^2 + \sigma^2{}_R = 39.7 \quad \cdots\cdots (4)$$

$$D(\bar{x}) = 6.3\% \quad \cdots\cdots (5)$$

where $\sigma_b^2 = (5.9)^2$, $\sigma_\omega^2 = (2.0)^2$, $\sigma_R^2 = 0$, $\sigma_M^2 = (1.9)^2$

On the other hand, when 13 times' sampling of three test pieces are carried out from 1900 m^3

concrete volume all through the construction term at the same sampling rate of one test per $150\,m^3$, $\nu(\bar{x})$ and $D(\bar{x})$ become thus. $\nu(\bar{x}) = 9.2$, therefore $D(\bar{x}) = 3.0\%$ however $\sigma_b = 8.5\%$

From these it is understandable that the precision of estimation in accordance with JIS method depends on the size of concrete volume to be placed and also on the number of sampling.

3.2. Precision and Sample Size

The frequency of sampling exercised was far larger than JIS method. What is the difference between the results, then? Following sample size might have been taken in various application of JIS method.

Case A n = 60 most tightened application Case C n = 21 slightly reduced application
Case B n = 39 normal application Case D n = 13 most reduced application

From 157 datum, 10 times random samplings of prescribed numbers were tried and those results are shown in Table-6～9. Randomization of sampling was done in accordance with tables of random number given in JIS Z9031. Even in the case of small sample size, the goodness of fit for $N(331, 28^2)$ was confirmed from the result of Chi-square test and this indicated complete

	n	$\bar{x}$	σ	MIN.	MAX.	R	CHI SQUARE χ_0^2	0.01	0.05
1	13	334	25.0	297	377	80	8.0	Not	Not
2	13	321	21.0	287	350	63	20.5	″	Sig.
3	13	333	30.0	293	389	96	12.9	″	Not
4	13	339	28.0	292	376	84	13.0	″	″
5	13	334	24.5	288	363	75	15.3	″	″
6	13	327	23.5	292	367	75	4.7	″	″
7	13	321	32.0	274	369	95	20.8	″	Sig.
8	13	342	27.0	299	389	90	7.0	″	Not
9	13	341	30.5	293	391	98	9.2	″	″
10	13	330	31.5	287	376	89	22.0	Sig	Sig.
Ave.	—	332±7.5	27.5±3.7	—	—	—	—	—	—

Table-6 10 Times random sampling of 13 samples from 157

	n	$\bar{x}$	σ	MIN.	MAX.	R	CHI SQUARE χ_0^2	0.01	0.05
1	21	328	29.5	280	372	92	10.1	Not	Not
2	21	333	28.0	287	376	89	17.2	″	Sig.
3	21	327	30.0	285	389	104	10.7	″	Not
4	21	328	26.5	285	389	104	16.1	″	″
5	21	341	20.0	297	372	75	8.2	″	″
6	21	342	24.0	293	391	98	10.6	″	″
7	21	326	22.5	293	362	69	15.3	″	″
8	21	336	26.0	285	372	87	9.6	″	″
9	21	330	25.5	285	376	91	4.7	″	″
10	21	325	24.5	292	377	85	13.9	″	″
Ave.	—	332±6.0	25.5±3.1	—	—	—	—	—	—

Table-7 10 Times random sampling of 21 samples from 157

	n	$\bar{x}$	σ	MIN.	MAX.	R	CHI SQUARE χ_0^2	0.01	0.05
1	39	332	29.0	282	389	107	12.8	Not	Not
2	39	330	27.5	288	391	103	12.9	″	″
3	39	331	28.0	274	.391	117	6.7	″	″
4	39	333	25.5	287	385	98	12.9	″	″
5	39	333	27.0	285	389	104	6.1	″	″
6	39	325	27.5	274	389	115	9.2	″ .	″
7	39	334	30.0	290	389	99	19.1	″	Sig.
8	39	331	29.0	280	385	105	12.1	″	Not
9	-39	333	25.0	· 274	388	114	3.4	″	″
10	39	336	27.5	285	391	106	20.3	″	Sig.
Ave.	—	332±2.9	27.5±1.5	—	—	—	—	—	—

Table-8 10 Times random sampling of 39 samples from 157

	n	$\bar{x}$	σ	MIN.	MAX.	R	CHI SQUARE χ_0^2	0.01	0.05
1	60	328	28.0	274	391	117	14.3	Not	·
2	60	330	27.0	274	389	115	8.0	″	″
3	60	329	24.5	280	388	108	2.9	″	″
4	60	331	. 25.0	280	391	111	10.4	″	″
5	60	333	26.5	280	391	111	3.8	″	″
6	60	334	26.0	280	389	109	9.1	″	″
7	60	337	24.0	292	388	96	7.8	″	″
8	60	337	28.5	285	389	104	19.2	″	″
9	60	339	27.0	274	391	117	3.9	″	″
10	60	333	27.0	280	388	108	3.8	″	″
Ave.	—	333±3.7	26.5±1.5	—	—	—	—	—	—

Table-9 10 Times random sampling of 60 samples from 157

Notes n: Sample size $\bar{x}$: Mean Concrete Strength of the sample
Min: Minimum Strength Max.: Maximum Strength
σ: Standard Deviations of the Sample Strength
R: Range of the Strength 0.01: Chi square value $(9, 0.01) = 21.7$
x_0^2: Chi square value of null Hypothesis on the fittness for Normal Distribution
0.05: Chi square value $(9, 0.05) = 16.9$
Not.: Not significant Sig.: Significant

randomness of the sampling.

Sample size n	relative standard error	
	Practical value (from 10 times random sampling)	theoretical value
n=10	14.9 %	22.0 %
n=13	14.1	18.5
n=21	14.1	14.0
n=30	9.0	12.5
n=39	5.8	11.0
n=50	7.5	10.0
n=60	6.1	9.0

Table-11 Relationship between sample size and relative standard error in the estimation of population standard deviation

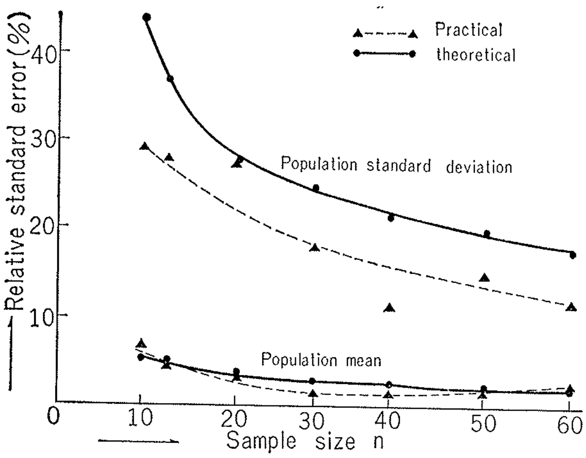


Fig.-3 Errors in the estimation of population mean and standard deviation (in the probability 95%)

Sample size n	relative standard error	
	practical value (from 10 times random sampling)	theoretical value D(x̄)/E(x̄)
n=10	3.17 %	2.67 %
n=13	2.26	2.35
n=21	1.83	1.89
n=30	0.91	1.54
n=39	0.90	1.36
n=50	1.06	1.20
n=60	1.11	1.09

Table 10 Relationship between sample cize and relative standard error in the estimation of population mean

Table-10 and 11 show the error variance of estimation of population mean and standard deviation between the trial observation and finite population. Error variance in population mean estimation is given theoretically in the function of sample size as follows.

Relative standard error

$$\frac{D(\bar{x})}{E(\bar{x})} = \frac{\sigma}{\sqrt{n}\,\mu} \quad \cdots\cdots\cdots\cdots\cdots \quad (6)$$

where n: sample size,

μ = population mean

The theoretical relationship between sample size and error variance in the estimation of population standard deviation was obtained from the figure given by J. A. Greenwood and M.M. Sandomire. Practical values are well in accordance with theoretical ones and Fig.-3 was obtained by plotting these results (in the probability of 95%).

Error variance of population mean estimation varies little with the decrease of sample size, but that of population standard deviation varies considerably. In the case of n = 39 which is the normal application of JIS, the biggest range of misunderstanding of lower control limit becomes about 15 kg/cm^2, although 8 kg/cm^2 in the case of n = 157.

4. CONCLUDING REMARKS

Through this investigation we could confirm that the high strength concrete used in the construction work came up almost to the objective quality level. Concurrently we cleared up the relation between statistical meanings of the datum and sampling methods. However, this is merely the one instance and we intend to develop the other application.