Inelastic Earthquake Response of Reinforced Concrete Buildings

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鉄筋コンクリート構造物の弾塑性地震応答解析

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概　要

鉄筋コンクリート部材のくり返し応力下での剛性低下を考慮した履歴曲線を定義し，これに基づいて鉄筋コンクリート構造物の地震応答について述べる。最初に鉄筋コンクリート構造物の強震時の挙動を知るために，EL-CENTRONS, 八戸十勝沖NS地震波に対する一質点系の塑性応答スペクトルを示し，最大変位について弾性スペクトル，降伏変位と比較した。次に，更に詳細な塑性応答時の挙動を知るために，各部材の critical section に塑性バネを挿入し，そのバネのモーメント～回転角の関係を用いて，フレーム自体の剛性変化を評価しながら応答を求める解析法を示し，その解析法に基づいて，10階建コンクリート構造物の応答解析例を示した。

Abstract

The realistic force-displacement relationsh of a reinforced concrete member which fails in bendiug under repeated loading is introduced. The response spectra of a one-mass system are calculated and discussed. Secondly, the analytical wethod used for non-linear response calculation of reinforced concrete frames is presented. In this method rigid-inelastic cotation springs which reflect the hysteresis are assumed at the critical section of each member. As an sxample, the response of a 10-storied reinforced concrete frame subjected to earthquake motion is discussed.

Notation

A:	area of reinforced concrete member
a:	shear span
a_t:	area of tensile reinforcement
B:	width of reinforced concrete member
D:	depth of reinforced concrete member
d:	distance from extreme compression fiber to centroid of tension reinforcement
E:	Young's modulus
F_c:	compressive strength of concrete
G:	modulus of shear rigidity
K:	absolute stiffness of member, I/l　$I=$ moment of inertia
K_1, K_2:	stiffness of inelastic springs at the ends (1) and (2)
l:	length of member
M_1, M_2:	bending moments at the ends (1) and (2)
M_{cr}:	cracking moment

M_Y:	yield moment
M_s:	mass
N:	axial force
n:	ratio of Young's modulus
R:	joint translation angle
p_t:	ratio of tensile reinforcement a_t/BD
α_Y:	ratio of stiffness at yielding to elastic stiffness
β:	shear stiffness reduction coefficient
δ_Y:	deflection at yielding
$\eta \cdot \phi$:	additional yield zone supposed to be distributed into beam-column connection
ϕ:	diameter of reinforcement
${}_H\Theta_{max}$:	maximum rotation angle
${}_H\theta_Y$:	rotation angle at yield moment
κ:	shape factor relating to shearing stress distribution
λ_1, λ_2:	ratios of the length of rigid zone at the ends (1) and (2) to the length of member
$1/\rho_{cr}$:	curvature at cracking moment
$1/\rho_r$:	curvature at yield moment

INTRODUCTION

Inelastic behavior of the reinforced concrete structure is not governed by the simple elasto-plastic or bilinear hysteresis loop and even by curvilinear Ramberg-Osgood hysteretic relation. Because cracking of concrete, slip and yielding of reinforcement of beams and columns decrease profoundly the stiffness of the structure. In order to discuss the earthquake response of the reinforced concrete buildings, it is necessary to define the realistic model which recognizes the continually varying stiffness and energy-absorbing characteristics of the structure.

As for the inelastic response analysis of multi-storied buildings, they are usually modified into shear type model with multi-lumped mass. However, there is problem in the modification itself. Furthermore the response of this system is not sufficient to evaluate the inelastic behavior of each column and girder in the sense of ductility. The more complete analysis of the dynamic response of buildings could be done by evaluating the inelastic rotation of spring inserted at the each end of members.

1. HYSTERESIS OF REINFORCED CONCRETE MEMBERS

An example of hysteresis is shown in Fig. 1. The hysteretic characteristic of reinforced concrete members introduced herein is almost same as that of the reference (1). Its applicability to reinforced concrete members was assured by the experiment of specimens subjected to dynamic base motion. The hysteresis law in the reference was constructed originally from the data of the specimens under alternative loading when they failed in bending. Only difference between hysteresis law used herein and that of the reference is in the calculation of primary curve (Fig. 1a) which consists of three linear segments and is described in each case hereafter.

2. SPECTRAL RESPONSE

Many experiments of beams and columns with stubs or without stubs indicate that yield deflections obtained were usually much larger than the values calculated on the basis of the plastic theory of reinforced concrete. This may be due to the reduction of shear rigidity and bond characteristics between bar and concrete, and mainly due to slipage of the bar from the concrete face of stub if provided. And as α_Y, the ratio of the stiffness at yielding to the elastic stiffness, the following equation has been recommended in A.I.J. Standard (Ref. 2, Fig. 2).

$$\alpha_Y = (0.043 + 1.64npt + 0.043a/D + 0.33N/BDF_c)(d/D)^2 \quad (1)$$

In the spectral calculation followed, $\alpha_Y = 0.25$ is assumed in each case to take into account the reinforced concrete characteristics. The crack level is just assumed to be a half or quarter of yield levels of $1G$ and $0.5G$, to see the difference of the responses. Thus the primary curves are determined. HACHINOHE 1968 NS and EL CENTRO 1940 NS, whose max. accelerations are standardized into $1G$, are used in the calculation. As shown in Fig. 3a and Fig. 3b, the difference between the spectrum to HACHINOHE and that to EL CENTRO is not remarkable. However the response to HACHINOHE of structure with natural period longer than 0.4 sec. gives larger displacements. The response displacement is generally smaller in the case of structure with higher crack level. However the difference of the crack level scarcely affects response deflection in the structure with short natural period, because their response is much larger than yield deflection. The nonlinear response values of the structures with natural period longer than 0.8 sec. do not exceed the yield displacement and are comparable to the linear response. However the response of the structure with shorter natural period than 0.6 sec. gives larger displacement than the elastic value. Especially the structure with $0.5G$ in yield acceleration shows remerkable increase in displacement. Accordingly larger ductility is required for the structure with comparatively low strength to the base exciting motion.

3. EQUATION OF INELASTIC STRUCTURAL FRAME AND RESPONSE ANALYSIS

3.1. Equation of Member Stiffness

Taking into account the cracking of concrete along the member due to bending moment directly in the dynamic response analysis of frames requires much time for calculation and we do not have enough data to make it realistic. In order to utilize the data obtained hitherto and make calculation simple, rigid-plastic springs are concenptually inserted in the members which may reflect the particular characteristics of reinforced concrete.

These springs are assumed to be located at the critical sections and, in the case of aseismic design of structures, at the each end of the members (Fig. 4). Completely rigid zone is also assumed at the beam-column connection in this analysis. The deflection of these members subjected to bending as shown in Fig. 4 can be expressed by the slope deflection method as follows;

$$\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} = \frac{6EK}{(\alpha'_A + r')(\alpha'_B + r') - (\alpha'_{AB} - r')^2} \begin{bmatrix} (\alpha'_B + r')(\alpha'_{AB} - r') - c \\ (\alpha'_{AB} + r')(\alpha'_A + r') - c' \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ R \end{Bmatrix} \quad (2)$$

in which

$$\alpha'_A = 2\{(1-\lambda_1)^3 - \lambda_2^3\} + \{(1-\lambda_1)^2 S_1 + \lambda_2^2 S_2\}$$
$$\alpha'_{AB} = \{1 - 3(\lambda_1^2 + \lambda_2^2) + 2(\lambda_1^3 + \lambda_2^3)\} + \{\lambda_1(1-\lambda_1)S_1 + \lambda_2(1-\lambda_2)S_2\}$$
$$\alpha'_B = 2\{(1-\lambda_2)^3 - \lambda_1^3\} + \{(1-\lambda_2)^2 S_2 + \lambda_1^2 S_1\}$$
$$C = \alpha'_B + \alpha'_{AB} \qquad C' = \alpha'_A + \alpha'_{AB}$$
$$S_1 = 6EK/K_1 \qquad S_2 = 6EK/K_2$$
$$\gamma' = 6EK\kappa(1 - \lambda_1 - \lambda_2)/\beta \cdot G \cdot A \cdot l$$

The shear deformation of beam-column connecting part due to horizontal load can be considered by the method described in the reference (3), if required.

3.2. Spring Stiffness K_1, K_2

The variation of flexural stiffness due to cracking of concrete and yielding of reinforcement is controlled by spring stiffness K_1, K_2, which represents the moment for unit angle of rotation of the spring. Linear behavior is expressed by $K_1, K_2 = \infty$, and complete plastic behavior is expressed by $K_1, K_2 = 0$. Multi-linear relationship or complicated hysteresis can be expressed by continually varying K_1, K_2. Generally spring stiffness can be assumed to be rigid up to the cracking moment. Once end moment exceeds the cracking moment, it is necessary to transfer the non-linear moment-curvature relationship along the member into the moment-rotation relationship of the spring. This may be done by assuming the state of equal clock-wise moment at each end of member (Fig. 5), and then following expression is obtained.

$$ {}_H\theta_Y = \frac{(1 - M_{cr}/M_Y)(0.5 - \lambda)(1/\rho_Y - M_Y/M_{cr} \cdot 1/\rho_{cr})\{1.5 - 3\lambda - (1 - M_{cr}/M_Y)(0.5 - \lambda)\}l}{3(1 - 2\lambda)} + \frac{\eta \cdot \phi \cdot 1/\rho_Y\{0.5\eta \cdot \phi + (0.5 - \lambda)l\}}{(0.5 - \lambda)l} $$

In equation (3), it is assumed that yielding curvature $1/\rho_r$ extends uniformly over the length equal to η bar-diameters into beam-column connection. Thus, the primary curve for the spring is defined, and the hysteresis law is applied to the moment-rotation relationship of spring in the same manner as described in section 2 (Fig. 6).

3.3. Response Analysis

The stiffness of complete frame may be obtained from the stiffness of each member by techniques of matrix structural analysis. With use of this stiffness of complete frame, the dynamic response equation is solved.

4. RESPONSE OF THE 10 STORIED REINFORCED CONCRETE BUILDING

4.1. Design

First suppose the building to be consisting of infinite number of bays with equal span subjected to horizontal force. This kind of building may be simplified into the frame with girders supported at mid span by rollers as shown in Fig. 7.

As an example this frame with 10 stories is designed by the Japanese code in which the lateral force coefficient is 0.2 up to 16 m in height and 0.01 has to be added at every next 4 m in height. The weight to be considered at each floor level is assumed as 30 ton. The bending moment and shear due to vertical load are not considered in this analysis. The following values of material properties are used. The strength of concrete: 210 kg/cm^2, the yield stress of reinforcement: 3500 kg/cm^2 for girders and 3000 kg/cm^2 for columns.

Tensile reinforcement ratios are 0.43–0.87% for columns except the ratio at the bottom end of the first story column (1.86 %), and 0.58–1.40% for girders. The axial stress of column of the first story is 46.9 kg/cm^2. The first two natural periods are $T_1 = 1.103$ sec., $T_2 = 0.399$ sec. (Table 1).

4.2. Response

Based on the designed members, the primary curve was calculated for each inelastic spring. In this calculation the assumptions were made as follows; the stress-strain relationships of concrete is expressed by an exponential function for the calculation of the yield moment and corresponding curvature. The length of additional yield zone $\eta \cdot \phi$ in eq (2) is $10 \cdot \phi$.

Acceleration and displacement at each floor are calculated for this frame with use of linear acceleration method. Two types of earthquake motion, EL CENTRO 1940 NS and HACHINOHE

1968 NS except that the maximum accelerations are changed into 300 gal and 450 gal. Only internal damping 5% is assumed in the equation.

4.2.1. Calculation Results

The maximum value of acceleration, total displacement and story deflection are shown in Fig. 10 and some examples of time history of acceleration and story deflection are indicated in Fig. 11.

The maximum accelerations of each floor attained in the inelastic response are smaller than those in the elastic one. The maximum total displacements in inelastic response to HACHINOHE are larger than those to EL CENTRO. It has to be marked, nonlinear response can be smaller compared with elastic response in every total displacements as shown in the case of EL CENTRO 300 gal. However in the case of the structure with rather long natural period there may not be remarkable difference between the total top displacements in nonlinear response and those in linear response.

It is interesting to note that the maximum values of the deflection, the story displacement, in inelastic response to HACHINOHE are larger than those to EL CENTRO and vice versa in elastic response.

The maximum story deflection are observed at the 8th story in inelastic response to every type of excitations. When $\ddot{y}_{0\max}$ is 300 gal the maximum deflections were 2.79 cm to HACHINOHE, and 2.09 cm to EL CENTRO. When $\ddot{y}_{0\max}$ is 450 gal the maximum deflections were 5.58 cm and 4.38 cm respectively. The ratio of the maximum deflection of the 8th story at 300 gal excitation to that at 450 gal is 2.06 to HACHINOHE and 2.10 to EL CENTRO. This phenomenon indicates that once a certain story failed, the deflection is concentrated to that story. These characteristics of the structure can not be predicted from the elastic response which indicates the maximum story displacements are almost equal except upper two stories. Further more such smooth distribution of the maximum story displacements around the 8th story may not be obtained from the non-linear response of shear type model with multi-lumped mass.

As seen in Fig. 11, the periodicity of the structure is seen around 5 second in elapsed time when the maximum response values is attained. To 450 gal of ground excitation, the periodicity is 1.52 sec. to HACHINOHE and 1.36 sec. to EL CENTRO. This periodicity is far longer than elastic natural period of 1.1 sec. This indicates again that the damages are more severe to HACHINOHE earthquake.

4.2.2. The Inelastic Rotation of Springs and Ductility

The plastic hinges had been formed as shown in Fig. 8. By EL CENTRO 300 gal, four hinges were formed at the end of girders of the 8th and the 9th floor, but no hinge was formed at the columns. By EL CENTRO 450 gal, hinges were formed at the end of girders from the 6th to the 9th floor and the bottom end of the column in the 7th story and the top end of column in the 9th story. The moment-rotation relationship at the bottom end of the column in the 7th story is shown in Fig. 9.

The ductility factor has been discussed in the nonlinear response of shear type model of multi-lumped mass. But it is impossible to attribute it to the ductility of beams or columns involved. This frame analysis has same kind of difficulties, although ductility in terms of hinge rotation ${}_H\theta_{\max}/{}_H\theta_Y$ is obtained. Because in columns, the inflection points move from the mid height of story. In order to be referred by the experimental data of beams and columns, approximate evaluation of ductility in terms of displacement of the column with fixed end, is made as follows. The spring characteristics are kept as described in section 3 and the inflection point at maximum response is assumed in the calculation of yield and maximum deflections δ_Y, $\delta'_{\max}$.

The ductility factors of columns thus obtained are indicated in Table 2. The ductility factors of springs (Table 2) are larger than those because ${}_H\theta_Y$ and ${}_H\theta_{\max}$ don't contain the elastic rotation. It seems easy to design the column for the ductility indicated in Table 2 to the 450 gal earthquake motion.

4.2.3. Unloading and Redistribution of Bending Moment

An example of bending moment redistribution of the frame subjected to increasing static loads is indicated in Fig. 12 in which P is increased from 49.80 ton to 52.80 ton. The solid line indicates the bending moment distribution just before the first yield hinge is formed at No. 1. When load is increased monotonously after first hinge formation, unloading of moments occurs as indicated by the dotted line in Fig. 12a). The most significant unloading of moment is observed at the top of the column in the first story. Moment decreases from 80.61 t•m to 28.29 t•m in spite of increasing of shear force from 73.04 ton to 85.36 ton. If load is increased much more, the moment at the top of this column begins to increase again. As discussed above, once yield hinge is formed at beams the peculiar behavior may be observed hereafter.

CONCLUSION

1. The inelastic response spectra of one degree of freedom which reflects the realistic restoring force of characteristics of the reinforced concrete are calculated. The calculation shows that larger displacement has to be anticipated in the structure with natural period less than 0.6 sec., compared with that of elastic response. This tendency is emphasized when the yield strength level of the structure decreases.
2. The inelastic analysis method of reinforced concrete frames is presented. This is done by providing the conceptual rigid-inelastic spring at each end of the members which reflects the characteristics of reinforced concrete member.
3. As an example the response of the 10 storied building is calculated and the maximum response, ductility, redistribution of bending moment and so forth are discussed.

REFERENCE

(1) Takeda, T., Sozen, M. S., and Nielsen, N. N., "Reinforced concrete response to simulated earthquakes" Proc. ASCE vol. 96, No. ST-12, Dec. 1970

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(3) Naka, T., Kato, B., Aoyama, H., et al., "Recent researches of structural mechanics", Contributions in honour of the 60th birthday of Procf. Tsuboi, Y., 1968

(4) Yoshioka, K., Takeda, T., "Inelastic earthquake response of one storied frame", Proc. St. No. 42 1962 Kanto Branch of AIJ

(5) Omote, Y., Takeda, T., Moritaka, I., Yoshioka, K., "Experiment and research on the response of the model structure under impact loading (Part 1)—Reinforced concrete frames" Proc. the 3rd Japan Earthquake Engineering Symposium, 1970.

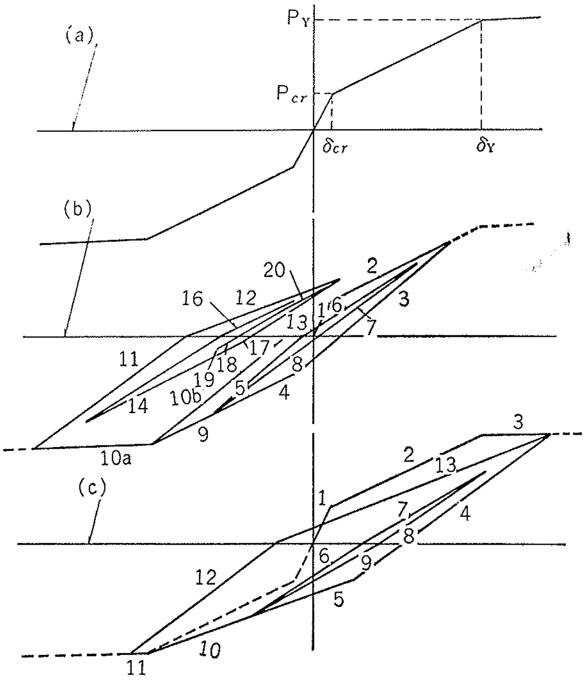


Fig.-1 EXAMPLE OF ASSUMED FORCE-DISPLACEMENT RELATIONSHIP

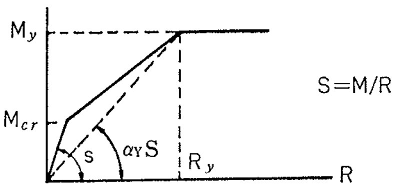


Fig.-2 STIFFNESS REDUCTION AT YIELDING

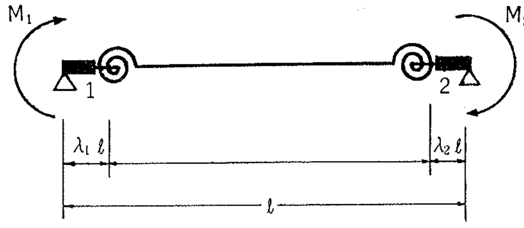


Fig.-4 INSERTED SPRING

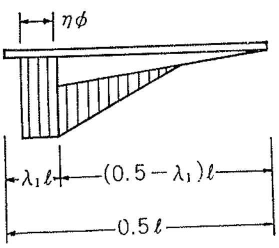


Fig.-5 MOMENT-ROTATION ANGLE RELATIONSHIP

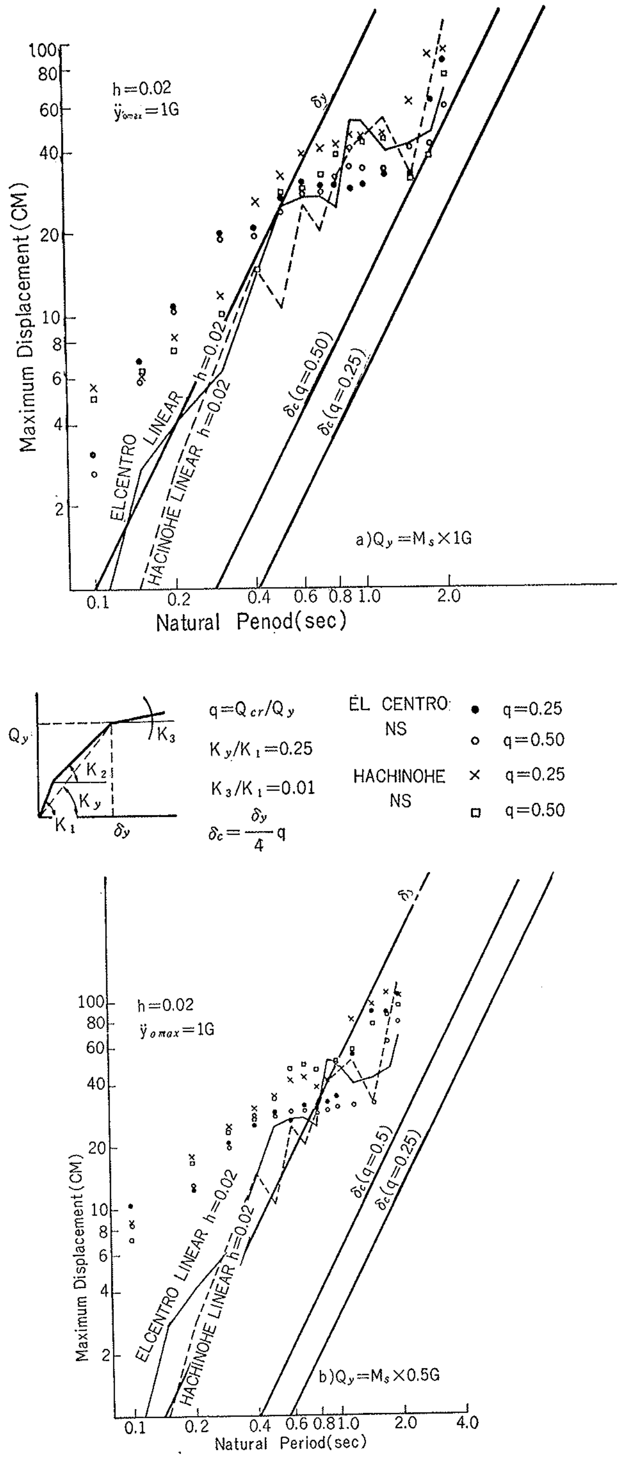


Fig.-3 SPECTRAL RESPONSE

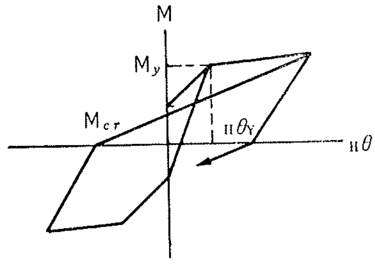
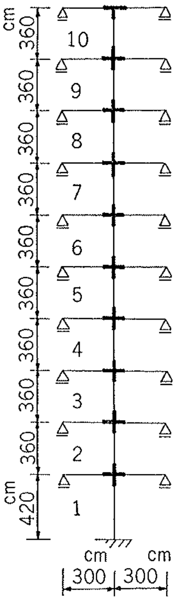


Fig.-6 CURVATURE DISTRIBUTION



	Story	Section
Column	10 ~ 6	B×D=70×70cm I=20.01×10⁵cm⁴
	5 ~ 1	B×D=80×80cm I=34.13×10⁵cm⁴
Girder	R ~ 8	B×D=40×70cm I=11.43×10⁵cm⁴
	7 ~ 5	B×D=40×80cm I=17.07×10⁵cm⁴
	4 ~ 2	B×D=40×90cm I=24.30×10⁵cm⁴

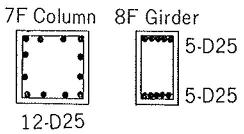


Fig.-7 EXAMPLE OF REINFORCED CONCRETE FRAME

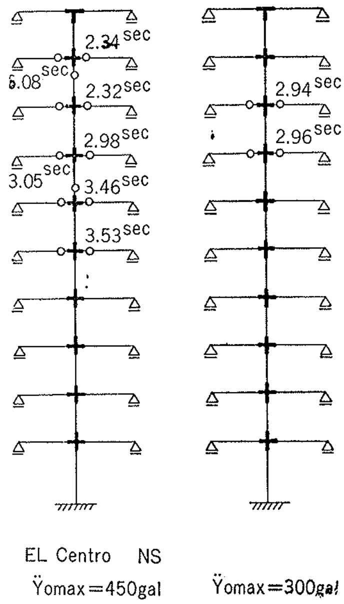


Fig.-8 FORMATION OF INELASTIC HINGE

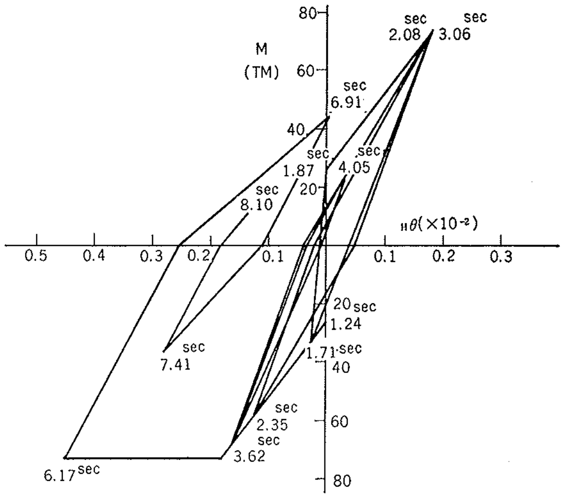


Fig.-9 MOMENT-ROTATION RELATIONSHIP OF THE SPRING AT THE BOTTOM OF THE 7TH STORY COLUMN

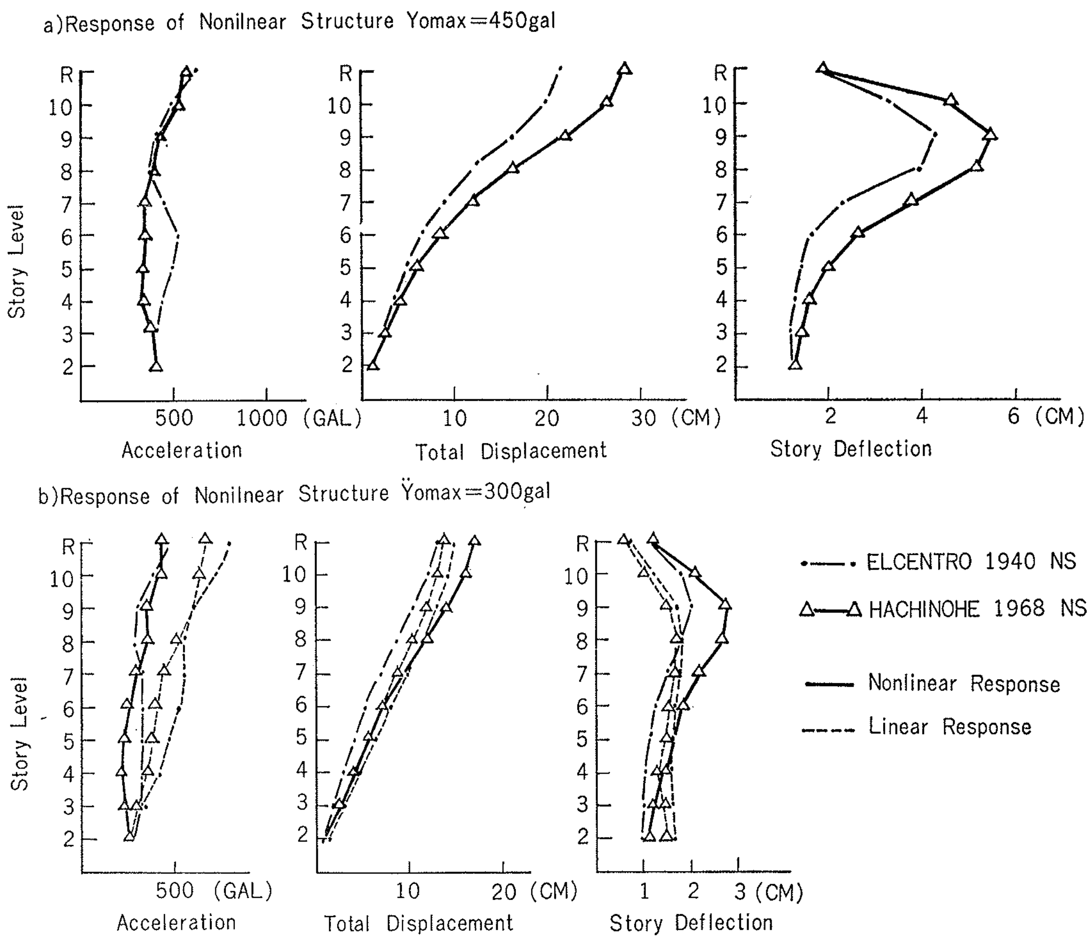
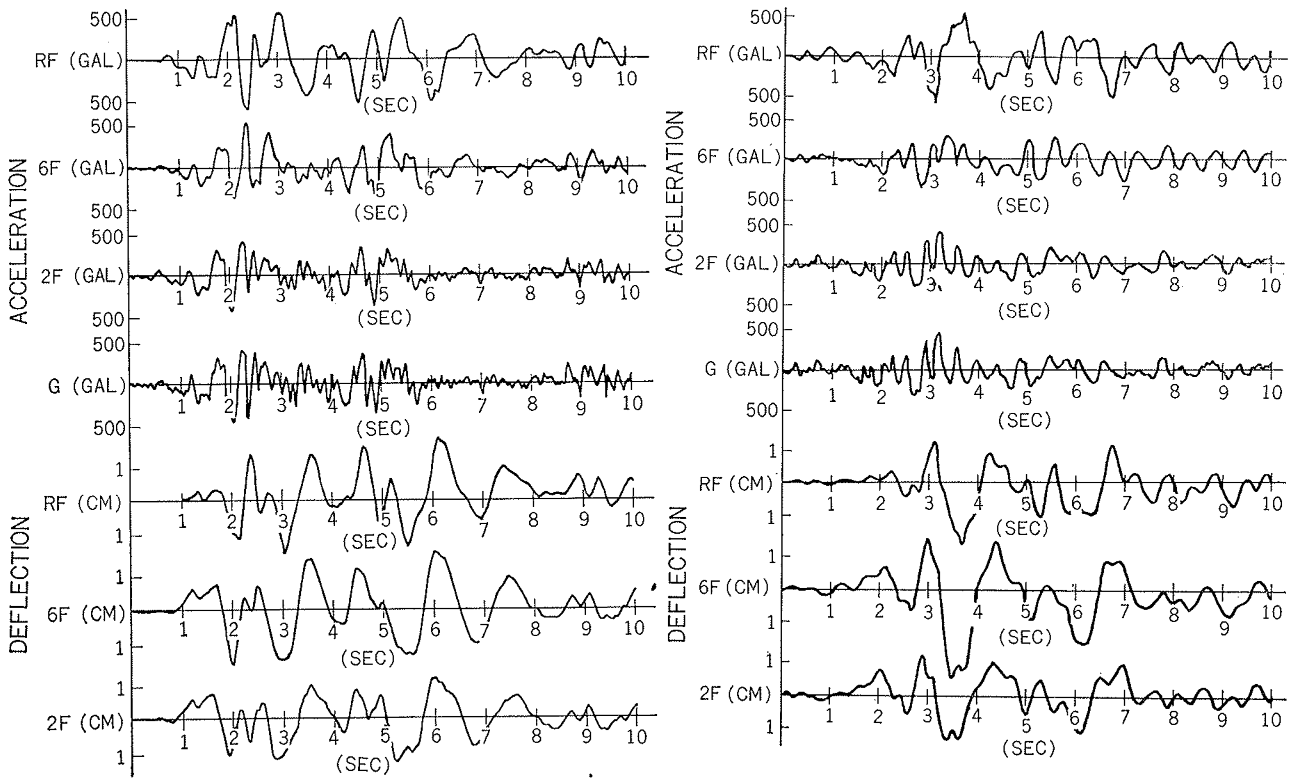


Fig.-10 THE MAXIMUM VALUE OF ACCELERATION, TOTAL DISPLACEMENT AND STORY DEFLECTION

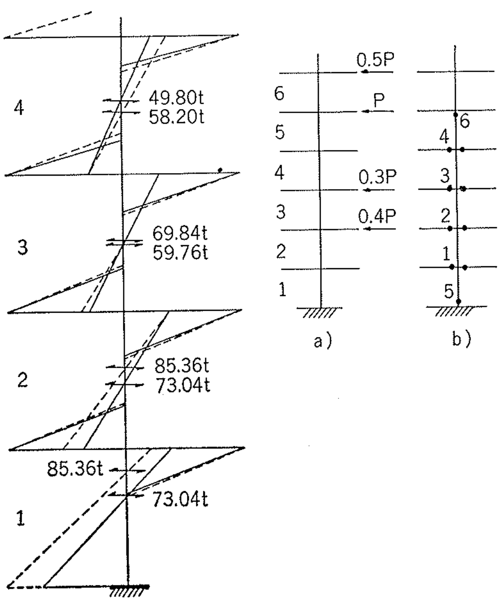


a) EL CENTRO 450 GAL b) HACHINOHE 450 GAL

Fig.-11 CALCULATED ACCELERATION AND STORY DEFLECTION AT THE ROOF, THE 6TH AND THE 2ND STORY LEVEL

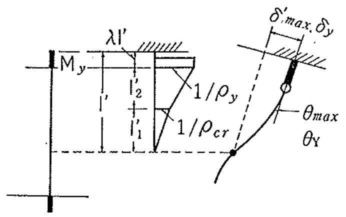
T_1 (sec)	T_2 (sec)	T_3 (sec)	T_4 (sec)
1.103	0.399	0.229	0.155

Table-1 NATURAL PERIOD OF THE FRAME



c)

Fig.-12 MOMENT REDISTRIBUTION UNDER MONOTONOUS STATIC LOADING



Story		Earthquake	δ_Y (cm)	δ'_{max} (cm)	$_H\theta_Y$ ($\times 10^{-2}$)	$_H\theta_{max}$ ($\times 10^{-2}$)	$\frac{\delta_{max}}{\delta_Y}$	$\frac{\delta_{max}}{_H\theta_Y}$
7	Bottom of columun	ELCENTRO NS 450gal	0.562	1.053	.183	.452	1.87	2.48
		HACHINOHE NS 450gal	0.536	0.915	.183	.398	1.71	2.18
9	Top of columun	ELCENTRO NS 450gal	0.835	0.978	.176	.309	1.16	1.75
		HACHINOHE NS 450gal	0.648	1.712	.176	.684	2.64	3.89

$$\delta_Y = \frac{1}{3}\times 1/\rho_{cr}\times\frac{M_Y}{M_{ax}}\times(1-\lambda)l'^2+\frac{1}{2}\times(1/\rho_Y-1/\rho_{cr}\frac{M_r}{M_{cr}})\times(l'_1+\frac{2}{3}l'_2)$$
$$+1/\rho_Y\times 10\times\phi\times\{(1-\lambda)l+5\phi\}$$

$$\delta'_{max}=\delta_Y+({}_H\theta_{max}-{}_H\theta_Y)(1-\lambda)l'$$

Table-2 DUCTILITY FACTOR OF COLUMNS